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Nonlinear amplification of adhesion forces in interleaved books

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Abstract

Abstract It is nearly impossible to separate two interleaved phonebooks by pulling their spines. The very slight force exerted by the outer sheets of the assembly is amplified as the exponential of the square of the number of sheets, meaning that even a small number of sheets can create a highly resistant system. We present a systematic and detailed study of the influences of the normal external force and the geometrical parameters of the booklets on the assembly strength. We conclude that the paper-paper adhesion force between the two outer sheets, on the order of a few mN, is the one amplified by the interleaved-book system. The two-phonebook experiment—which has attracted the attention of students and the non-scientific public all around the world as an outstanding demonstration of the strength of friction—appears to also be a spectacular macroscopic manifestation of the microscopic coupling of friction and adhesion.

Solid friction is a classic phenomenon relevant to both daily life and sophisticated engineering applications, but at the most fundamental level, questions still remain [1]. In their pioneering works, Coulomb and Amontons developed what are now called the Coulomb–Amontons laws of friction. In particular, they introduced a static friction coefficient, which is the ratio between the traction force required at the onset of sliding and the normal force between two solids. Understanding the molecular mechanisms of friction, however, took a considerable amount of time and effort. Tabor was the first to identify how the adhesive junction between microscopic surface asperities could be responsible for one of the most non-trivial characteristics of the friction coefficient: it is independent of the apparent contact area between the two sliding surfaces [2]. More recently, much attention has focused on understanding friction at the micro- and nanoscales [3–5], in biology, [6,7] and in meta-materials [8]. Among the open questions of tribology remains an understanding of the link between adhesion and friction [9–16]. Biological systems such as a gecko's toes show a precise coupling of the normal and tangential forces due to adhesion, which is particularly relevant in understanding the mechanisms of detachment [17–20]. More generally, it has now been shown that the exact coupling between normal and tangential forces is not limited to the gecko foot, but is a ubiquitous and general phenomenon that holds for the adhesive pads of all terrestrial climbing animals that have been tested thus far [21]. In engineering, another example of the importance of tuning adhesion in order to control friction is the haptic device, in which an electric field enables modification of the adhesion between fingers and a touch screen in order to control their friction, thus generating a sensitive mechanical stimulation [22,23]. Systems that exhibit many frictional contacts could be used to better understand how adhesion can modify global macroscopic performance. From a physical perspective, much insight has been gained from the study of granular materials in which a small amount of humidity can strongly affect the mechanical properties of the system [24–26], as well as from examinations of common materials in everyday use, such as braids [27], knitted fabrics [28,29], and interlocked chains or fibers [30].

Another example of these common, yet puzzling, systems is the popular demonstration of the strength of friction in which two phonebooks are interleaved page-by-page and pulled apart by their spines [31,32]. Based on experimental testing of controlled paper assemblies, a simple model was presented in 2016 that captures

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the force necessary to separate the books as a function of their interleaving distance and the number of sheets [32]. The main idea of the model is that the minute friction force T^* exerted by the outer sheet of the assembly on the sheets below is amplified because of the inherent angle present in the interleaved geometry; this induces a conversion of the operator's traction into a supplementary normal force—and thus additional friction. This is a mechanism similar to the well-known amplification of the tension force created with a capstan. The pulling force T exerted on the whole assembly was then linked to the separation distance d between the booklets and the number $2M$ of sheets in each booklet thusly [32]:

$$T = 2MT^* \sqrt{\frac{\pi}{4\alpha}} \exp(\alpha) \mathrm{erf}\left(\sqrt{\alpha}\right), \tag{1}$$

where α is the *Hercules number* defined as $\alpha = 2\mu\epsilon M^2/d$, μ is the friction coefficient, and ϵ is the thickness of one sheet.

Alarcon *et al.* treated the parameters μ and T^* as fitting parameters [32]. However, T^* was a parameter with a direct impact on the mechanical properties of the assembly, and was found to be around 10 mN in most of the experiments that were reported.
The proportionality relationship that is predicted by Eq. 1 between T^* and T remains to be validated, and the origin of T^* must be investigated. Moreover, the physical origin of T^* needs to be elucidated. Several forces could contribute significantly to T^*: the weight of the outer sheet or anything that is attached to it; the bending elastic force, due to the angle of the interleaved assembly; or the intermolecular interaction between the last sheet and the sheet below, which is the essence of adhesion.

To further characterize the role and origin of T^*, we carried out a systematic experimental study by carefully interleaving two paper stacks sheet by sheet. We placed a rigid cover on either side of the assembly in order to avoid any significant bending of the external sheet. The booklets and the two covers were clamped in metallic jaws and fixed vertically into a traction-force machine (Adamel Lhomargy DY32). The total traction force T was then measured (with an accuracy of 0.1 N) as a function of the separation distance d (measured with an accuracy of 10 µm) between the clamp and the contact area (see Fig. 1). The two booklets were pulled apart at a constant and tunable speed varying from 1 mm/min to 10 mm/min. Each booklet was composed of the same number of identical sheets of paper (Inacopia Office$^{\mathrm{TM}}$, 80 g/m^2, "silky touch"), of length L and width l that were varied among the experiments. The thickness of a sheet was ϵ =0.1 mm and was kept constant through all experiments. Special attention was given to the initial separation distance d_0 of the booklets, which was measured after they were clamped into the traction-force machine. Small d_0 values led to unreproducible measurements of T^*. Indeed, when d_0 becomes smaller than 15 mm for this typical number of sheets, the sheets' angles become large and the small-angle approximation done in [32] is no longer valid.

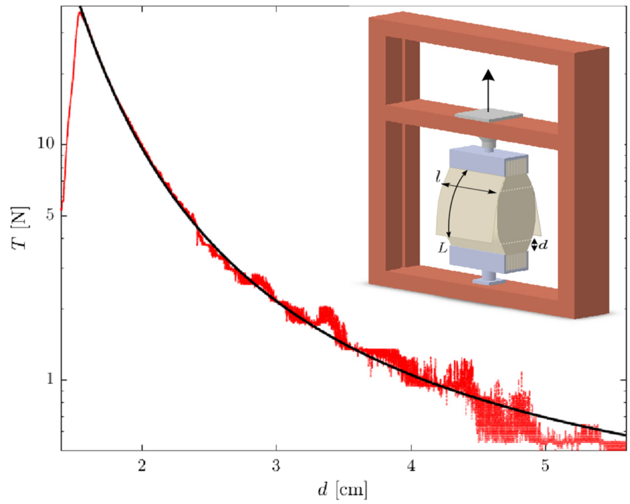

Fig. 1 Traction force exerted on the two-booklet assembly as a function of interleaved distance, measured for two booklets of $2M = 30$ sheets each, whose dimensions are L=16 cm, l=21 cm, and ϵ=0.1 mm. The solid line represents the best fit to 1. The calculated fitting parameters are $\mu = 2.4$ and T^*=4 mN. The traction force T is recorded by the traction-force machine in which the booklets are clamped (see inset) and is equal to zero when the books separate

Using the previously described setup, a first experiment was designed to drastically change the boundary condition by adding an external load on the cover. A spacer and a rigid cover holding an adjustable load were added on top of the first cover, on each side of the assembly (see Fig. 2). A soft polytetrafluoroethylene (PTFE) sheet was added between the spacer and the first cover in order to minimize the stick-slip of the external load. The spacer allowed us to maximize the normal load exerted on the assembly by increasing the angle θ and thus the horizontal component of the weight-induced tension in the second cover.

Using various external masses m in this design, force-displacement curves were recorded. Using a linear least-squares method, those $T(d)$ curves were well fitted by Eq. 1 with μ and T^* as free parameters. The measured values of T^* allowed us to rescale all the curves, as plotted in the inset of Fig. 3, thus confirming that the model developed without an external load is still valid. Furthermore, as shown in Fig. 3, T^* was found to depend on the external mass and thus on the effective external load. Indeed, when a small weight is added, T^* clearly changes by more than two orders of magnitude. More precisely, T^* varies linearly with the applied load. The best linear fit for the overall data can be expressed as $T^* = (0.32 \pm 0.01)W$ and is shown as a dashed line in Fig. 3. To examine this dependency further, we can consider the force balance on the outer mass at rest: $\overrightarrow{F_T} - \overrightarrow{N} + \overrightarrow{W} = \overrightarrow{0}$, where $\overrightarrow{F_T}$ is the tension force in the second cover on which the mass is fixed, $\overrightarrow{N}$ is the normal force exerted by the mass on the booklet assem-

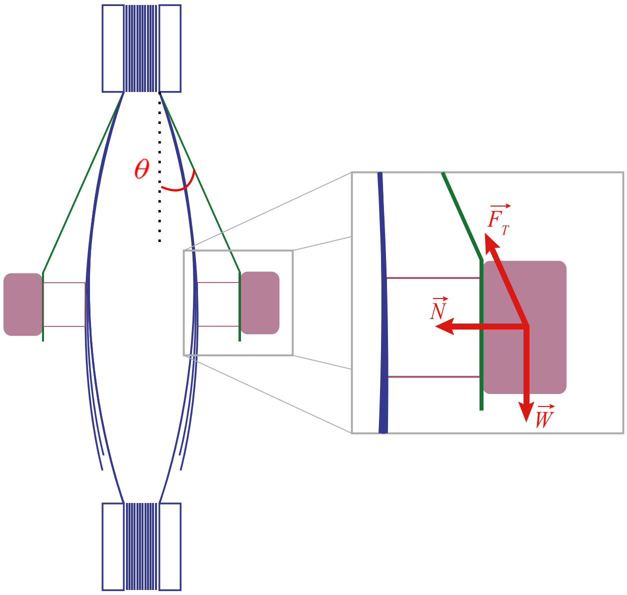

Fig. 2 Side view of the two-booklet assembly with a tunable external load. The assembly is covered on each side by two additional sheets (see inset): the first (in green) is rigid cardboard and the second (in blue) is composed of soft PTFE. Two masses weighing m grams are fixed to the first cover, which is attached to the upper clamp. To maximize the force, a spacer is inserted between the masses and the soft cover and fixed to the rigid cover

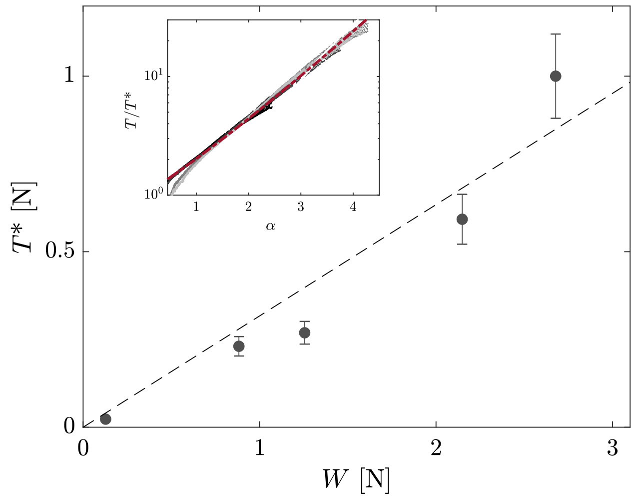

Fig. 3 Boundary friction force T^* **as a function of external weight.** The dashed line corresponds to the best linear fit, with a slope of 0.32 ± 0.1. The error in T^*, computed by repeating the experiment, was 11%, and this value was used to generate error bars in the figure. The external weight is the weight of the entire mounting device with the tunable load, plus the weight of the second cover. The inset shows T/T^* as a function of α for the different masses tested, as well as a best fit (dashed red) line for Eq. 1, with μ and T^* as free parameters

bly, and $\overrightarrow{W}$ is the weight (see Fig. 2). A combination of the vertical and horizontal projections—taking into account the angle θ between the rigid cover and the vertical axis—gives $N = W \tan\theta$. As T^* is the friction force on the outer sheet, Amontons-Coulomb law gives us $T^* = \mu N$ at the onset of the motion, which leads to $T^* = \mu W \tan\theta$. In the experiments represented in Fig. 3, the measured θ was around 15° and the mean value of μ was $\mu = 1.07$; with these, we obtain $T^* = (0.29 \pm 0.1)W$. Note that the small dependency of the friction coefficient with the normal load has been reported for this experiment in [32]. This is in good agreement with the experimentally measured slope in Fig. 3. Therefore, it seems reasonable to conclude that in this case T^* originates from and is proportional to the external mass. More generally, our results extend the validity of Eq. 1 and the conclusion of Alarcon *et al.* that the total traction force T results from a frictional and geometrical amplification of T^* [32]. Paper is a complex material, and the friction coefficient is a phenomenological quantity whose value depends on different parameters [33]. For paper-paper interactions, the friction coefficient depends on a wide range of phenomena occurring at different scales, such as interlocking asperities and adhesion forces [34]. More specifically, the kinetic friction coefficient of paper is known to depend on the normal load [35] (as it does for metal-metal interfaces [36]) and to increase with a reduction in the normal load [32,37]; this can lead to values higher than 1, such as those found by fitting 1 to our experimental data.

In order to determine the origin of the boundary force T^* when no external mass is added, several experiments were performed with booklets composed of sheets of different dimensions, but keeping the total number of sheets in the assembly at $4M = 60$. In a first experiment, the sheet width l varied from 5 cm to 21 cm while the length L remained constant at 16 cm (see Fig. 4). We observed that T^* varied linearly with l. In the second experiment, the sheet length L varied from 3 to 25 cm with a fixed width l of $l = 12$ cm (see inset of Fig. 4). In this case, T^* was found to be scattered but essentially independent of L.

A first possible origin for T^* could be the weight of the first cover. Using the characteristic grammage of the 80 g/m^2 paper, this would imply $T^*/l = \mu W \tan\theta / l \approx$ 1 mN/m, which is 40 times smaller than the measured value (see Fig. 4). The weight of the cover is thus not the main origin of the measured T^* values in our experiments, but could become important with books with hard covers.

A second possible origin of T^* is the elastic bending force. Indeed, the first cover is curved, and thus exerts a restoring elastic force on the assembly. The force required to bend a sheet depends on the boundary conditions of clamping, but scales as $E\epsilon^3 l M \epsilon / x_0^3$, with E the Young's modulus of paper and x_0 the distance between the clamping point and the application point of the bending force [38]. Typical values of Young's modulus for paper are between 3 and 5 GPa [39]. This force is proportional to the width of the page, similar to the T^* measured here. However, the distance x_0 is not obvious. It must be in between d and L; the latter can be rejected

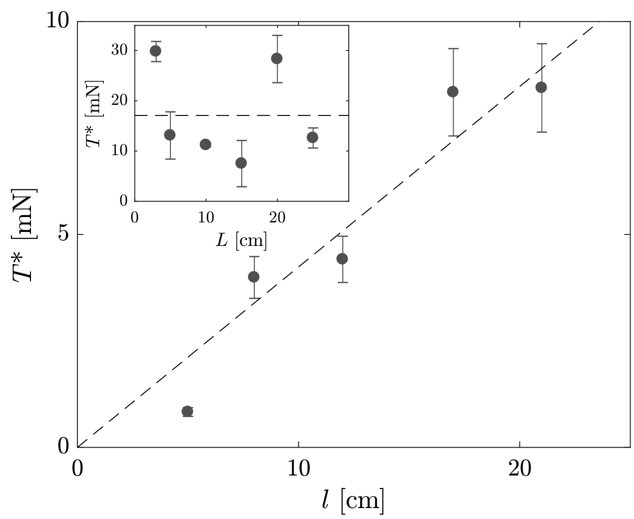

Fig. 4 Boundary friction force as a function of sheet width. Using booklets with a constant length L=16 cm and various widths l, force-displacement curves were recorded and fitted to Eq. 1, with T^* and μ as free parameters. The best linear fit (represented as a dashed line) has a slope of 42 mN/m. The inset shows the values of T^* for booklets with the same width l=12 cm and different lengths L

due to the fact that we do not observe any dependency of T^* with respect to L. Although it is the most intuitive hypothesis, we cannot have $x_0 = d$ either, since it would mean that the bending force changes during the experiment and we would thus not be able to fit our results with a single value of T^*. Similarly, any intermediate value between d and L would be expected to depend on d, and should thus be rejected as well.

The last possible explanation takes into account the adhesion energy between the two outermost sheets. Indeed, it is well known that the adhesive peeling force between two surfaces is proportional to the contact width l and the energy release rate G [40–42]. This is due to the fact that, during detachment, only a small fraction of the strip adhering to the substrate is subjected to the applied load, because the typical magnitude of the deformation field near the contact line is on the same order as the thickness of the adhesive layer. Since we are interested in a quasi-static situation, the dissipation processes can be discounted and the energy release rate G can be identified as the work of adhesion, w_{A}. For a paper-paper symmetrical interfacial rupture, we have $w_{\mathrm{A}} = 2\gamma_{\mathrm{p}}$ where γ_{p} is the interfacial tension. Considering rough contacts, we can estimate the effective adhesion energy as $w_{\mathrm{A,eff}} = 2\gamma_{\mathrm{p}}A/A_0$, where A/A_0 is the fraction of the apparent area A_0 effectively involved in the contact. The interfacial tension γ_{p} of hydrophobic cellulose fibers is ~ 29.6 mN/m [43]; if we combine this with the mean value of μ=2.5 as a fitting parameter for the experiments using low masses in Fig. 4, the effective adhesion force per unit of width is comparable to the value of $N/l = T^*/(\mu l)$ =16.8 mN/m obtained in Fig. 4 if we assume $A/A_0 \approx 0.28$, which is a possible value for rough surfaces [44]. The values for the work of adhesion obtained by this estimate seem higher than expected. However, Electrostatic forces, which are known to play an important role in paper printing, and which are adhesive too, could explain these values.

In conclusion, by investigating in a systematic manner the effects of an external mass and the geometrical parameters of the booklets on the tearing force, and specifically on the boundary force T^*, we were able to identify the origin of T^*. The linear dependency of T^* on the external mass provided further confirmation that Eq. 1, and its underlying model based on the frictional and geometrical amplification of the boundary force, captures the phenomenon well. This further suggests that the resistance of an assembly of interleaved booklets could be finely controlled by an external load; the uses of such an assembly thus go beyond schoolroom physics demonstrations to applications as a mechanical transistor, with the outer mass acting as the gate terminal. This type of device has recently attracted a great deal of attention from the soft robotic community [45–48].
We also observed a linear increase in T^* with the booklets' width. Combined with the independence of T^* from the books' length, this observation allows us to show that, in the absence of an external load, the main contribution to T^* cannot be either the weight of the cover or the bending of the cover sheet. As a result, we suggest that the physical origin of the force exerted by the cover is its adhesion to the sheet. Further experiments using the same system of interleaved sheets composed of different materials—and thus with lower or higher adhesion forces—would provide an interesting way of tuning the strength of the assembly.

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Author contribution statement

R.T. did the experiments. All authors interpreted the results and developed the models. R.T., F.R. and C.P. wrote the manuscript and all the authors read and corrected the manuscript.

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