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WEAK ADHESIVE JUNCTIONS IN THE PRESENCE OF INTERMOLECULAR INTERACTIONS

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Polymer adhesive has been the subject of many theoretical and experimental studies in the last few years [1]. Certain weak adhesive junctions [2] may be described by a mechanical model where: (i) the opening rate dh/dt vanishes when the stress σ is below a certain threshold value σ^* and (ii) for $\sigma>\sigma^*$, the opening rate increases with the stress as $dh/dt = Q^{-1}(\sigma-\sigma^*)$ where Q is a friction coefficient. This model, called in the literature the Model II, has been explored in detail [3][4]. Intermolecular interactions, which give rise to the thermodynamic work of adhesive W, have been recently incorporated into this model in the limit of zero crack velocity [5][6]. In this paper we generalise these studies to finite crack velocities. In particular, we calculate the length of the junction L as well as the fracture energy G.

Consider the problem of a mode I semi-infinite crack embedded in a linear purely elastic material. For plane strain conditions the normal stress distribution $\sigma(x)$ and the crack displacement u(x) are respectively given by [7]

$$\sigma(x) = \frac{K}{\sqrt{2\pi x}} \quad x>0; \quad \sigma(x)=0 \qquad x<0 \tag{1}$$

and

$$u(x) = \frac{2(1-\nu)}{\mu} K \sqrt{\frac{-x}{2\pi}} \quad x<0; \quad u(x)=0 \qquad x>0 \tag{2}$$

where μ is the shear modulus, ν the Poisson ratio and K the stress intensity factor. The fracture energy per unit area G is given by Irwin's equation [8]

$$G = \frac{1-\nu}{2\mu} K^2. \tag{3}$$

In a cohesive zone model the singular stress (1) is relaxed by inelastic deformation in a zone directly ahead of the crack (see Fig. 1). In this region ($0<x<L$) the elastic field may be described in terms of a source function $\Phi(x)$ defined by [9][3]

$$\sigma(\mathrm{x}) = \mu \int_0^{\mathrm{x}} dy \Phi(y)(\mathrm{x}-y)^{-1/2} \tag{4}$$

and

$$u(\mathrm{x}) = 2(1-\nu) \int_{\mathrm{x}}^{L} dy \Phi(y)(y-\mathrm{x})^{1/2}. \tag{5}$$

It can be shown [10] that (4) and (5) are equivalent to the standard formulation [11]

$$u(\mathrm{x}) = \frac{2(1-\nu)}{\mu} K \sqrt{\frac{L-\mathrm{x}}{2\pi}} - \frac{1-\nu}{\pi\mu} \int_0^L dy \sigma(y) \ln \left| \frac{\sqrt{L-\mathrm{x}}+\sqrt{L-y}}{\sqrt{L-\mathrm{x}}-\sqrt{L-y}} \right|. \tag{6}$$

At large distance (|x| >> L), (4) and (5) reduce to (1) and (2) with

$$\frac{K}{\sqrt{2\pi\mu}} = \int_0^L dy \Phi(y). \tag{7}$$

The mechanical behaviour of the junction is described [5][12] by the following law relating the rate of opening dh/dt=2du/dt and the normal stress σ (Model II)

$$\frac{dh}{dt} = Q^{-1}(\sigma - \sigma^*) \qquad \sigma > \sigma^*; \quad \frac{dh}{dt} = 0 \qquad \sigma < \sigma^*. \tag{8}$$

where Q is the friction coefficient of the junction and σ^* is a threshold stress. Note that the opening rate is continuous at $\sigma = \sigma^*$.

We now consider the problem of a steadily growing crack with translational velocity V. Inserting (4) and (5) into (8) we obtain the fundamental equation

$$\Delta_\lambda \Phi(\mathrm{x}) = \frac{\sigma^*}{\mu} \tag{9}$$

where Δ_λ is the following linear integral operator

$$\Delta_\lambda \Phi(\mathrm{x}) \equiv \int_0^{\mathrm{x}} dy \Phi(y)(\mathrm{x}-y)^{-1/2} - \lambda \int_{\mathrm{x}}^{L} dy \Phi(y)(y-\mathrm{x})^{-1/2} \tag{10}$$

and

$\lambda = V/V^*$ a reduced velocity with $V^* = \mu/[2(1-\nu)Q]$.

Equation (9) is a non-homogeneous linear integral equation for the source function $\Phi(\mathrm{x})$. It must be supplemented by the boundary condition

$$h_f = 2u(x = 0) \tag{11}$$

where h_f is the terminal value of the junction opening. Using (5), (11) can be rewritten as

$$\frac{h_f}{4(1-\nu)} = \int_0^L dy\Phi(y)y^{1/2}. \tag{12}$$

We have also the conditions that
(i) in the absence of connectors G reduces to W, (13)
(ii) in the zero-rate limit ($V \to 0$) G reduces to $G_0 = W + \sigma^* h_f$ [6]. (14)

The solution of (9) can be written as

$$\Phi(x) = \Phi_H(x) + \Phi_M(x) \tag{15}$$

where $\Phi_H(x)$ and $\Phi_M(x)$ satisfy respectively

$$\Delta_\lambda \Phi_H(x) = \frac{\sigma^*}{\mu} \tag{16}$$

and

$$\Delta_\lambda \Phi_M(x) = 0. \tag{17}$$

The source function $\Phi_H(x)$ was derived by Fager et al. [4],

$$\Phi_H(x) = \frac{\sigma^*}{\mu} \frac{1}{B(\frac{1}{2}-\varepsilon,\ \frac{1}{2}+\varepsilon)} x^{-\left(\frac{1}{2}+\varepsilon\right)} (L-x)^{\varepsilon} \tag{18}$$

where ε is related to the reduced velocity λ by

$$\tan(\pi\varepsilon) = \lambda. \tag{19}$$

In (18), B(x,y) denotes the beta function $B(x,y) = \Gamma(x)\,\Gamma(y)/\Gamma(x+y)$ where $\Gamma(x)$ is the gamma function [13]. The source function $\Phi_M(x)$, which originates from the intermolecular forces, is given by

$$\Phi_M(x) = \left(\frac{W}{\pi\mu(1-\nu)}\right)^{1/2} \frac{1}{B(\frac{1}{2}-\varepsilon,\varepsilon)} L^{1/2} x^{-\left(\frac{1}{2}+\varepsilon\right)} (L-x)^{-1+\varepsilon}. \tag{20}$$

This form is obtained by solving (18) with the conditions (14) and (15). The dependence

$$\Phi_M(x) \propto x^{-\left(\frac{1}{2}+\varepsilon\right)}(L-x)^{-1+\varepsilon}$$

was already proposed by de Gennes and Troian in their investigation of the so-called Model I [14]. Knowing $\Phi(x) = \Phi_H(x) + \Phi_M(x)$, we can now calculate the length of the junction L, see (12), the stress intensity factor K, see (7) and finally the fracture energy G, see (3).

The length L of the cohesive zone can be determined by using the boundary condition (11). Inserting (15) into (12) we obtain

$$\frac{h_f}{4(1-\nu)} = \frac{\sigma^*}{\mu}\frac{B(1-\varepsilon,1+\varepsilon)}{B(\frac{1}{2}-\varepsilon,\frac{1}{2}+\varepsilon)}L + \left(\frac{W}{\pi\mu(1-\nu)}\right)^{1/2}\frac{B(1-\varepsilon,\varepsilon)}{B(\frac{1}{2}-\varepsilon,\varepsilon)}L^{1/2}. \tag{21}$$

Hence

$$\frac{L}{L_0} = \left[\frac{B(\frac{1}{2},\frac{1}{2}-\varepsilon)}{B(\frac{1}{2},\frac{1}{2})}\frac{\sqrt{1+\frac{\sigma^* h_f}{W}}+1}{\sqrt{1+\frac{\sigma^* h_f}{W}\frac{B\left(1+\varepsilon,\frac{1}{2}-\varepsilon\right)}{B\left(1-\varepsilon,\frac{1}{2}+\varepsilon\right)}}+1}\right]^2 \tag{22}$$

where L_0 denotes the zero-rate limit

$$L_0 = \frac{\pi}{4(1-\nu)}\frac{\mu h_f^2}{W}\frac{1}{\left(\sqrt{1+\frac{\sigma^* h_f}{W}}+1\right)^2}. \tag{23}$$

At low velocities (V<<V*) we obtain

$$L = L_0\left(1+\frac{4\ln 2}{\sqrt{1+\frac{\sigma^* h_f}{W}}}\frac{V}{V^*}+\dots\right) \qquad \frac{V}{V^*} \ll 1. \tag{24}$$

At higher velocities (V>>V*) two cases must be considered depending on the value of the parameter $\sigma^* h_f/W$
(i) if $\sigma^* h_f/W >> 1$ then

$$L \cong \frac{1}{2(1-\nu)}\frac{\mu h_f}{\sigma^*}\frac{V}{V^*} \qquad \frac{V}{V}^* \gg 1 \tag{25}$$

(ii) if $\sigma^* h_f/W << 1$ then

$$L \cong \frac{\pi}{16(1-\nu)}\frac{\mu h_f^2}{W}\left(\frac{V}{V^*}\right)^2 \qquad 1 \ll \frac{V}{V^*} \ll \frac{W}{\sigma^* h_f} \tag{26}$$

$$L \cong \frac{1}{2(1-\nu)}\frac{\mu h_f}{\sigma^*}\frac{V}{V^*} \qquad 1 \ll \frac{W}{\sigma^* h_f} \ll \frac{V}{V^*}$$

Figure 2 represents the behaviour of L/L_0 as a function of the reduced velocity V/V^* for three values of the parameter $\sigma^* h_f/W$.

The stress intensity factor K can be determined by using (7). Inserting (15) into (7) we have

$$\frac{K}{\sqrt{2\pi\mu}} = \frac{\sigma^*}{\mu}\frac{B(\frac{1}{2}-\varepsilon, 1+\varepsilon)}{B(\frac{1}{2}-\varepsilon, \frac{1}{2}+\varepsilon)}L^{1/2} + \left(\frac{W}{\pi\mu(1-\nu)}\right)^{1/2}. \tag{27}$$

Substitution of (22) into (27) yields

$$K^2 = \frac{2\mu}{1-\nu}\left[W + \sigma^* h_f \frac{B(1+\varepsilon, \frac{1}{2}-\varepsilon)}{B(1-\varepsilon, \frac{1}{2}+\varepsilon)}\right]. \tag{28}$$

Using Irwin's equation (3) we obtain the fracture energy of the junction

$$G = W + \sigma^* h_f \frac{B(1+\varepsilon, \frac{1}{2}-\varepsilon)}{B(1-\varepsilon, \frac{1}{2}+\varepsilon)} \tag{29}$$

with the limiting behaviours

$$G = G_0\left(1 + \frac{4\ln 2}{\pi}\frac{1}{1+\frac{W}{\sigma^* h_f}}\frac{V}{V^*} + \dots\right) \qquad \frac{V}{V^*} \ll 1$$

$$G \cong \sigma^* h_f \frac{\pi}{2}\frac{V}{V^*} \qquad \frac{V}{V^*} \gg 1 \tag{30}$$

where G_0 is the zero-rate limit: $G_0 = W + \sigma^* h_f$ [6].

Figure 3 represents the behaviour of $(G-W)/G_0-W)$ as a function of the reduced velocity V/V^*. This function is independent of the parameter $\sigma^* h_f/W$, see (29). Equation (29) indicates that at any velocity the fracture energy is a linear combination of the thermodynamic work of adhesion and the pull-out work.

In this paper we have investigated the properties of the so-called Model II taking into account molecular interactions. This model may be used to describe the process of chain pull-out and the relation between chain pull-out and interface toughness in the adhesion of elastomers [15][16]. Our main results are expressions (22) and (29) for the length L of the junction and for the fracture energy G as a function of the reduced velocity V/V*, the thermodynamic work of adhesion W and the zero-rate limit of the pull-out work $\sigma^* h_r$

Note that in our model the stress $\sigma(x)$ ahead of the cohesive zone tip becomes much larger than σ^* as one approaches x = L. Nevertheless, the junction remains closed for x > L for one must first reach a sufficiently large stress to overcome the attractive intermolecular forces and to separate the two sides of the junction. Only then can the opening process described by (8) take place.

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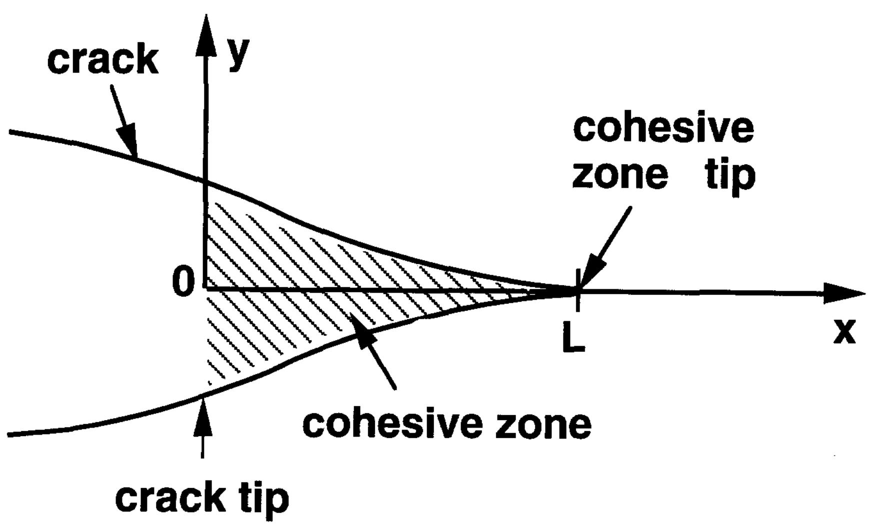

Figure 1. A global view of the cohesive zone.

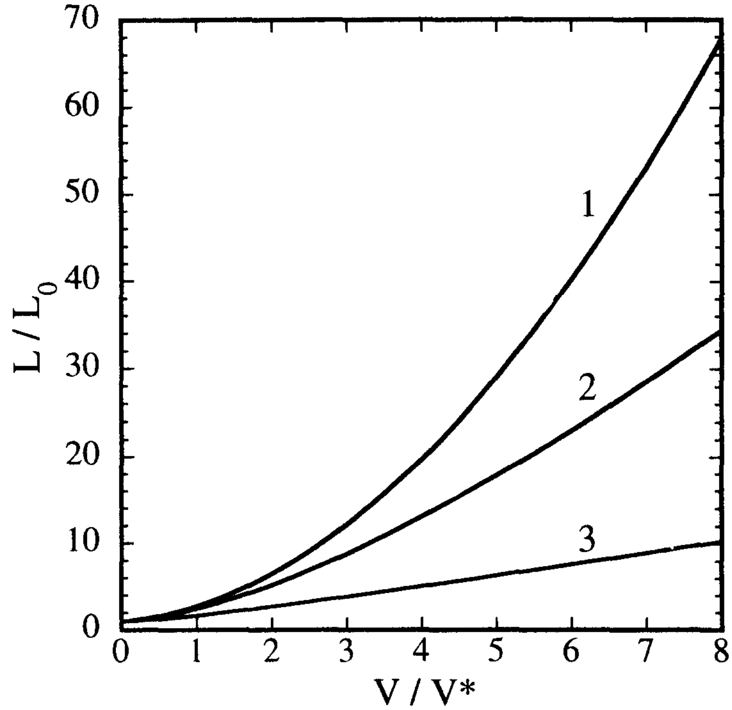

Figure 2. Plots of L/L_0 versus V/V^* for different values of the parameter $\sigma^* h_f/W$: curve 1, $\sigma^* h_f/W=0.01$; curve 2, $\sigma^* h_f/W=0.25$; curve 3, $\sigma^* h_f/W=5$.

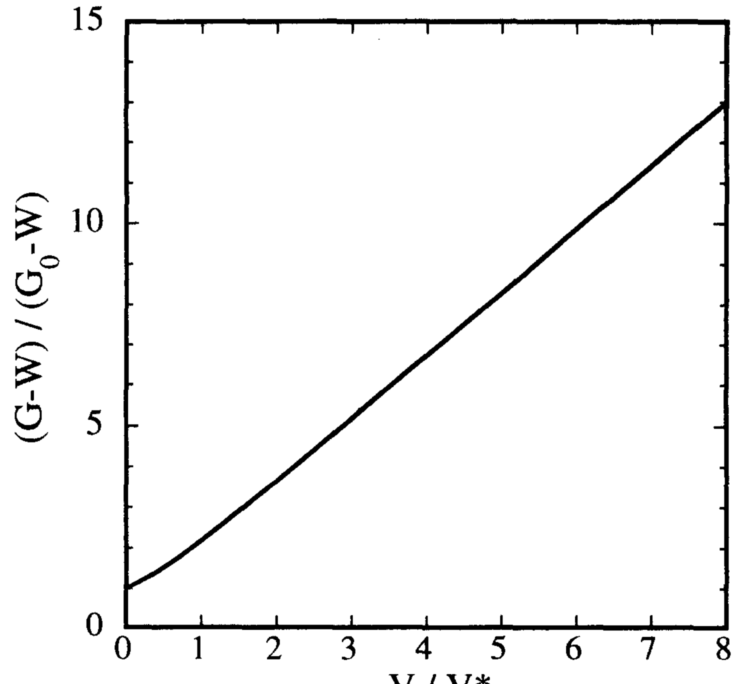

Figure 3. Plot of $(G-W)/(G_0-W)$ versus V/V^*.